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## Systematic design of high-performance hybrid feedback algorithms

Andrew Teel  
UNIVERSITY OF CALIFORNIA SANTA BARBARA

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Final Report

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| <b>14. ABSTRACT</b><br>The objective of this research is to develop tools for systematic design of hybrid feedback algorithms in a variety of settings. Hybrid feedback algorithms, which combine classical feedback and logic or other reset mechanisms, are of interest to the Air Force because of the high degree of autonomy and cooperation that future reconnaissance and fighter aircraft will maintain. Complex control and coordination algorithms for such vehicles, involving multiple mode switching and other high-level supervisory control architectures, give rise to complicated hybrid dynamical systems with behaviors that can be difficult to predict and understand. Building on previous tools for predicting the behavior of complex hybrid dynamical systems, the proposed research aims to provide systematic tools for shaping the behavior of complex hybrid dynamical systems. Our techniques will involve Lyapunov-based approaches, optimization-based model predictive control, and dissipativity-based results. |                                    |                                     |                                   |   |  |
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**FINAL REPORT**

**SYSTEMATIC DESIGN OF HIGH-PERFORMANCE  
HYBRID FEEDBACK ALGORITHMS**

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## Summary

The objective of this research was to develop tools for systematic design of hybrid feedback algorithms in a variety of settings. Hybrid feedback algorithms, which combine classical feedback and logic or other reset mechanisms, are of interest to the Air Force because of the high degree of autonomy and cooperation that future reconnaissance and fighter aircraft will maintain. Complex control and coordination algorithms for such vehicles, involving multiple mode switching and other high-level supervisory control architectures, give rise to complicated hybrid dynamical systems with behaviors that can be difficult to predict and understand. The proposed research aimed to provide systematic tools for shaping the behavior of complex hybrid dynamical systems.

One set of problems that provided motivation included stabilization, tracking, synchronization, and disturbance attenuation for physical systems where the kinematic variables evolve on a compact manifold. This setting is relevant for the feedback control of satellites and other airborne rigid bodies that can experience complete angular rotation. A second motivation involved tracking, synchronization, and disturbance attenuation for mechanical systems that experience collisions. A third motivation involved rejection of discontinuous disturbances using internal model techniques. Our techniques involved Lyapunov-based approaches, optimization-based techniques, and dissipativity-based results. For hybrid systems, we developed and employed backstepping design techniques, high-gain observer techniques, and internal model techniques, which parallel what is available for continuous-time systems. We also developed novel control ideas that have no analogue for continuous-time systems. Moreover, we began to develop analysis tools for stochastic discrete-time and hybrid systems and used randomness in hybrid control algorithms to achieve enhanced robustness properties.

## Research Publications

The research supported by this grant resulted in 24 journal papers, 25 refereed conference papers, and 2 book chapters. These publications are listed below.

### Journal papers

- J1. A.R. Teel, “A recurrence principle for stochastic difference inclusions”, *IEEE Transactions on Automatic Control*, vol. 60, no. 2, pp. 420-435, 2015.
- J2. Prieur, C.; Teel, A.R.; Zaccarian, L., “Relaxed persistent flow/jump conditions for uniform global asymptotic stability”, *IEEE Transactions on Automatic Control*, vol. 59, no. 10, pp. 2766-2771, 2014.
- J3. A.R. Teel, A. Subbaraman, and A. Sferlazza, “Stability analysis for stochastic hybrid systems: a survey”, *Automatica*, vol. 50, no. 10, pp. 2435-2456, 2014.
- J4. N. Noroozi, D. Nešić, and A.R. Teel, “Gronwall inequality for hybrid systems”, *Automatica*, vol. 50, no. 10, pp. 2718-2722, 2014.
- J5. A.R. Teel, J.P. Hespanha, and A. Subbaraman, “A converse Lyapunov theorem and robustness for asymptotic stability in probability”, *IEEE Transactions on Automatic Control*, vol. 59, no. 9, pp. 2426-2441, 2014.
- J6. Liberzon, D.; Nešić, D.; Teel, A.R., “Lyapunov-based small-gain theorems for hybrid systems”, *IEEE Transactions on Automatic Control*, vol. 59, no. 6, pp. 1395-1210, 2014.
- J7. N. Cox, L. Marconi, and A.R. Teel, “High-gain observers and linear output regulation for hybrid exosystems”, *International Journal of Robust and Nonlinear Control*, 24 (6), 1043-1063, 2014.
- J8. Teel, A.R.; Hespanha, J.P.; Subbaraman, A., “Equivalent characterizations of input-to-state stability for stochastic discrete-time systems,” *IEEE Transactions on Automatic Control*, vol.59, no.2, pp. 516-522, 2014.
- J9. Marconi, L.; Teel, A.R., “Internal model principle for linear systems with periodic state jumps”, *IEEE Transactions on Automatic Control*, vol.58, no.11, pp. 2788-2802, 2013.
- J10. Mayhew, C.G.; Teel, A.R., “Synergistic hybrid feedback for global rigid-body attitude tracking on  $SO(3)$ ”, *IEEE Transactions on Automatic Control*, vol.58, no.11, pp.2730-2742, 2013.
- J11. A. Subbaraman, A.R. Teel, “A Matrosov theorem for strong global recurrence”, *Automatica*, Volume 49, Issue 11, November 2013, Pages 3390-3395.

- J12. A. Subbaraman, A.R. Teel, “A converse Lyapunov theorem for strong global recurrence”, *Automatica*, Volume 49, Issue 10, October 2013, Pages 2963-2974.
- J13. S. Grammatico, A. Subbaraman, A.R. Teel, “Discrete-time stochastic control systems: a continuous Lyapunov function implies robustness to strictly causal perturbations”, *Automatica*, Volume 49, Issue 10, October 2013, Pages 2939-2952.
- J14. L. Fagiano, A.R. Teel, “Generalized terminal state constraint for model predictive control”, *Automatica*, Volume 49, Issue 9, September 2013, Pages 2622-2631.
- J15. D. Nešić, A.R. Teel, G. Valmorbida, L. Zaccarian, “Finite-gain  $L_p$  stability for hybrid dynamical systems”, *Automatica*, Volume 49, Issue 8, August 2013, Pages 2384-2396.
- J16. Teel, A.R., “A Matrosov theorem for adversarial Markov decision processes”, *IEEE Transactions on Automatic Control*, vol.58, no.8, pp.2142-2148, 2013.
- J17. C. G. Mayhew, A. R. Teel, “Global stabilization of spherical orientation by synergistic hybrid feedback with application to reduced-attitude tracking for rigid bodies”, *Automatica*, Volume 49, Issue 7, July 2013, Pages 1945-1957.
- J18. Forni, F.; Teel, A.R.; Zaccarian, L., “Follow the bouncing ball: global results on tracking and state estimation with impacts”, *IEEE Transactions on Automatic Control*, vol.58, no.6, pp.1470-1485, 2013.
- J19. Teel, A.; Forni, F.; Zaccarian, L., “Lyapunov-based sufficient conditions for exponential stability in hybrid systems”, *IEEE Transactions on Automatic Control*, vol.58, no.6, pp.1591-1596, 2013.
- J20. Mayhew, C.G.; Sanfelice, R.G.; Teel, A.R., “On path-lifting mechanisms and unwinding in quaternion-based attitude control”, *IEEE Transactions on Automatic Control*, vol.58, no.5, pp.1179-1191, 2013.
- J21. Heemels, W.P.M.H.; Donkers, M.C.F.; Teel, A.R., “Periodic event-triggered control for linear systems”, *IEEE Trans. on Automatic Control*, vol.58, no.4, pp. 847-861, 2013.
- J22. A. R. Teel, “Lyapunov conditions certifying stability and recurrence for a class of stochastic hybrid systems”, *Annual Reviews in Control*, Volume 37, Issue 1, April 2013, Pages 1-24.
- J23. C. Cai and A.R. Teel, “Robust input-to-state stability for hybrid systems”, *SIAM Journal on control and optimization* (2013) volume: 51 issue: 2 page: 1651-1678.
- J24. R.H. Gielen, A.R. Teel, M. Lazar, “Tractable Razumikhin-type conditions for input-to-state stability analysis of delay difference inclusions”, *Automatica*, Volume 49, Issue 2, February 2013, Pages 619-625.



### Conference papers

- C1. J.I. Poveda and A.R. Teel, “Flexible Nash seeking using stochastic difference inclusions”, *Proceedings of the American Control Conference*, 2015, to appear.
- C2. F. Forte, L. Marconi, A.R. Teel, “Robust nonlinear regulation: continuous-time internal models and hybrid identifiers”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, 4703-4708, 2014.
- C3. C.O. Saglam, A.R. Teel, and K. Byl, “Lyapunov-based versus Poincare map analysis of the rimless wheel”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, 1514-1520, 2014.
- C4. A.R. Teel, “Stochastic hybrid inclusions with diffusive flows”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, 3071-3076, 2014.
- C5. J.I. Poveda and A.R. Teel, “A hybrid seeking approach for robust learning in multi-agent systems”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, 3463-3468, 2014.
- C6. M. Hartman and A.R. Teel, “Adding resets to an average consensus algorithm to achieve practical input-to-output stability”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, 704-708, 2014.
- C7. S.J.L.M. van Loon, W.P.M.H. Heemels, and A.R. Teel, “Improved L2-gain analysis for a class of hybrid systems with applications to reset and event-triggered control”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, pp. 1221-1226, 2014.
- C8. A. Subbaraman and A.R. Teel, “A Krasovskii-LaSalle function based recurrence principle for a class of stochastic hybrid systems”, *Proceedings of the 53<sup>rd</sup> IEEE Conference on Decision and Control*, pp. 2310-2315, 2014.
- C9. A.R. Teel, “On sequential compactness of solutions for a class of stochastic hybrid systems”, *Proceedings of the American Control Conference*, pp. 4512-4517, 2014.
- C10. A.R. Teel, “On a recurrence principle for a class of stochastic hybrid systems”, *Proceedings of the American Control Conference*, pp. 4518-4523, 2014.
- C11. J.T. Isaacs, C. Magee, A. Subbaraman, F. Quitin, K. C. Fregene, A. R. Teel, U. Madhow, J.P. Hespanha, “GPS-optimal micro air vehicle navigation in degraded environments”, *Proceedings of the American Control Conference*, 1864-1871, 2014.
- C12. N. Cox, L. Marconi, and A.R. Teel, “Design of robust internal models for a class of linear hybrid systems”, *IFAC World Congress*, 19 (1), 1525-1530, 2014.

- C13. J. Liu and A.R. Teel, "Hybrid systems with memory: modeling and stability analysis via generalized solutions", *IFAC World Congress*, 19 (1), 6019-6024, 2014.
- C14. A.H. Brodtkorb, A.J. Sorensen, and A.R. Teel, "Increasing the operation window of dynamic positioned vessels using the concept of hybrid control", *ASME 33<sup>rd</sup> International Conference on Ocean, Offshore and Arctic Engineering*, 2014.
- C15. Heemels, W.P.M.H.; Borgers, D.P.; van de Wouw, N.; Nešić, D.; Teel, A.R., "Stability analysis of nonlinear networked control systems with asynchronous communication: A small-gain approach," *52nd IEEE Conference on Decision and Control*, pp.4631-4637, 2013.
- C16. Subbaraman, A.; Hartman, M.; Teel, A.R., "A stochastic hybrid algorithm for robust global almost sure synchronization on the circle: all-to-all communication," *52nd IEEE Conference on Decision and Control*, pp.600-605, 2013.
- C17. Grammatico, S.; Subbaraman, A.; Teel, A.R., "Discrete-time stochastic control systems: examples of robustness to strictly causal perturbations," *52nd IEEE Conference on Decision and Control*, pp.6403-6408, 2013.
- C18. Teel, A.R.; Goebel, R.; Morris, B.; Ames, A.D.; Grizzle, J.W., "A stabilization result with application to bipedal locomotion," *52nd IEEE Conference on Decision and Control*, pp.2030-2035, 2013.
- C19. Cox, N.; Marconi, L.; Teel, A.R., "Results on non-linear hybrid output regulation," *52nd IEEE Conference on Decision and Control*, pp. 2036-2041, 2013.
- C20. A.R. Teel. "Sure almost global vs. global almost sure synchronization on the circle: the virtues of stochastic hybrid feedback", *Proceedings of the 9<sup>th</sup> IFAC Symposium on Nonlinear Control Systems (NOLCOS 2013)*, pp. 217-222, Sept. 2013.
- C21. Liberzon, D.; Nešić, D.; Teel, A.R., "Small-gain theorems of LaSalle type for hybrid systems," *Decision and Control (CDC), IEEE 51st Annual Conference on*, pp.6825-6830, 2012.
- C22. W. Wang; Teel, A.R.; Nešić, D., "Averaging in singularly perturbed hybrid systems with hybrid boundary layer systems," *Decision and Control (CDC), IEEE 51st Annual Conference on*, pp.6855-6860, 2012.
- C23. Teel, A.R.; Hespanha, J.; Subbaraman, A., "Stochastic difference inclusions: results on recurrence and asymptotic stability in probability," *Decision and Control (CDC), IEEE 51st Annual Conference on*, pp.4051-4056, 2012.
- C24. Cox, N.; Marconi, L.; Teel, A.R., "Hybrid internal models for robust spline tracking," *Decision and Control (CDC), IEEE 51st Annual Conference on*, pp. 4877-4882, 2012.

C25. Jun Liu; Teel, A.R., "Generalized solutions to hybrid systems with delays," *Decision and Control (CDC), IEEE 51st Annual Conference on*, pp. 6169-6174, 2012.

#### Book Chapters

B1. M. Hartman, A. Subbaraman, A.R. Teel. Robust global almost sure synchronization on a circle via stochastic hybrid control, In *Control of cyber-physical systems: Workshop Held at Johns Hopkins University, April 2013*, Springer, pp. 3-21.

B2. F. Forni, A.R. Teel, and L. Zaccarian, "Reference mirroring for control with impacts", In *Hybrid Systems with Constraints*, pp. 213-255, Wiley, 2013.

## Research Accomplishments

*Control of systems with impacts:* Tracking control and stabilization of periodic orbits for mechanical systems with impacts has been a notoriously difficult problem. In the papers [J18] and [B2], with technical support from [J19], we introduce a novel “mirroring” control concept for tracking control in mechanical systems with impacts. In contrast to other results, the mirroring idea alleviates the need for high-gain feedback and achieves global tracking using output feedback and observers. In additional novel work, we used basic results on robustness of stability for hybrid dynamical systems to make new observations about control of walking robots in [C3] and [C18].

*Control of systems on manifolds:* Topological obstructions prevent robust, global asymptotic stabilization of a point or tracking of a trajectory on a compact manifold using classical control techniques. However, these obstructions can be overcome with hybrid feedback. Constructive techniques for hybrid global stabilization and tracking were generated in [J10], [J17], and [J20]. The key concept introduced here was the idea of synergistic potential functions, or synergistic Lyapunov functions, and the construction of synergistic hybrid feedbacks from these. Additional hybrid feedback algorithms were also applied to ocean vessels in [C14].

*Hybrid output regulation:* Classical output regulation principles describe how to incorporate internal models of smooth disturbances to reject them completely at a plant output. In work that we started under our previous AFOSR grant, we investigated how to incorporate hybrid models of discontinuous disturbances, or disturbances with discontinuous derivatives, to reject them completely at a plant output. That work continued under the current support, with constructive output regulators provided in [J7] and [J9], controllers for hybrid output regulation of nonlinear systems in [C2] and [C19], robustness principles in [C12], and the application of hybrid output regulation to the problem of spline tracking in [C24].

*Event-triggered control:* Event-triggered control is an emerging area of study, developed over the last decade, where control signal updates are made only when necessary. Assessment of the need to update a control signal can be made continuously, periodically, or intermittently. In each case, a hybrid dynamical systems model captures well the dynamical behavior of the overall system. However, the case of periodic assessment, especially for linear plans, is especially well suited for an accurate Lyapunov-function analysis employing linear matrix inequality conditions. Such analysis and synthesis tools were proposed in [J21] and [C7], the former having already been cited a remarkable 102 times according to Google Scholar.

*Multi-agent and networked systems:* Multi-agent systems involve complicated interactions between dynamic agents, in some problems cooperating to achieve a common goal, like estimation consensus or synchronization, and other times being non-cooperative, attempting to maximize their own payoff function. Some multi-agent systems algorithms are deterministic while others employ randomness to achieve or enhance robustness. We used extremum-seeking ideas and switched system ideas for

cooperative multi-agent systems in [C5], used reset concepts for robust consensus in [C6], and analyzed the role of randomness and asynchronous actions in robust Nash equilibrium seeking in non-cooperative games in [C1]. In [C15], we provided new analysis tools for asynchronous, nonlinear, networked control systems. We used stochastic hybrid control to achieve almost sure global synchronization on the circle under different criteria in [C16], [C20], and [B1].

*Hybrid systems theory:* Hybrid dynamical systems are used to model situations where the state sometimes changes continuously and other times changes instantaneously through jumps. Such models address systems with state resets, mechanical systems with impacts, logic-based switching between controllers, networked control systems, etc. Various stability analysis tools for hybrid systems have been developed over the last twenty years, and yet there are several aspects of stability theory for hybrid systems that are not yet suitably developed. Such tools are critical for certifying effective behavior of hybrid control systems. In the current period, we provided new sufficient conditions for global asymptotic stability [J2], a hybrid Gronwall lemma [J4], a Lyapunov function based small-gain theorem [J6], [C21], analysis tools for assessing finite-gain  $L_p$  stability [J15], sufficient conditions for exponential stability [J19], a complete characterization of input-to-state stability for hybrid systems [J23], and new results on averaging theory for hybrid systems [C22]. Moreover, we began to develop a general framework that handles hybrid systems with delays in [C13] and [C25].

*Stochastic discrete-time and hybrid systems:* Stochastic hybrid systems go beyond standard hybrid dynamical systems by permitting the evolution to be driven by random processes, providing even more modeling flexibility. Various types of stochastic hybrid systems have been studied for multiple decades. We generated a paper surveying this literature in [J3]. One aspect that is particular to all of these earlier results is that uniqueness of solutions is enforced. However, there are many compelling reasons to study stochastic systems that admit multiple solutions. One motivation comes from treating open systems with disturbances as closed systems with non-unique solutions. Another motivation comes from attempting to guarantee robustness of stability for systems with non-regular data. (For more information, see the references discussed herein.) These motivations exist even for discrete-time stochastic systems, where non-uniqueness is also not usually considered in the literature. Recognizing the importance of establishing a better understanding of stochastic discrete-time and hybrid dynamical systems, in particular for the development of effective stochastic, hybrid control algorithms, we published a wide variety of results on this topic. Our results on stochastic discrete-time (difference) inclusions include an invariance principle [J1], converse Lyapunov theorems [J5], [J12], equivalent characterizations of input-to-state stability [J8], relaxed sufficient conditions for asymptotic stability and recurrence [J11], [J16], robustness results for discontinuous stochastic discrete-time systems [J13], [C17], and an application to robust Nash equilibrium seeking in non-cooperative games [C1]. An overview of most of these theoretical developments is contained in [C23]. For hybrid systems with non-stochastic flows (continuous evolution) but stochastic jumps, we provided an extensive set of sufficient conditions for asymptotic stability in probability and recurrence in [J22]. We also developed a useful sequential compactness result [C9],

and an ensuing invariance/recurrence principle [C10], [C8]. We considered hybrid systems with both stochastic flows and stochastic jumps in [C4], including Lyapunov function based sufficient conditions for asymptotic stability in probability and recurrence. Many open problems remain in completing the story on stability theory for stochastic hybrid systems with non-unique solutions. Development of these tools will enhance our ability to control complex dynamical systems using randomness and logic-based, decision-making criteria.

### **Acknowledgment/Disclaimer**

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### **Personnel Supported**

*Faculty:* Dr. Andrew R. Teel (PI);

*Graduate Student Researchers:* Nicholas Cox, Matthew Hartman, Jorge Poveda

### **Transitions**

none

### **Lifetime honors/awards**

IFAC Fellow, 2010

IEEE Fellow, 2002

AACC Donald P. Eckman Award, 1999

SIAM Control and Systems Theory Prize, 1998

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**Abstract**

Control of systems with impacts: Tracking control and stabilization of periodic orbits for mechanical systems with impacts has been a notoriously difficult problem. We introduced a novel "mirroring" control concept for tracking control in mechanical systems with impacts. In contrast to other results, the mirroring idea alleviates the need for high-gain feedback and achieves global tracking using output feedback and observers. In additional novel work, we used basic results on robustness of stability for hybrid dynamical systems to make new observations about control of walking robots.

Control of systems on manifolds: Topological obstructions prevent robust, global asymptotic stabilization of a point or tracking of a trajectory on a compact manifold using classical control techniques. However, these obstructions can be overcome with hybrid feedback. We generated constructive techniques for hybrid global stabilization and tracking. The key concept introduced here was the idea of synergistic potential functions, or synergistic Lyapunov functions, and the construction of synergistic hybrid feedbacks from these. Additional hybrid feedback algorithms were also applied to ocean vessels.

Hybrid output regulation: Classical output regulation principles describe how to incorporate internal models of smooth disturbances to reject them completely at a plant output. In work that we started under our

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previous AFOSR grant, we investigated how to incorporate hybrid models of discontinuous disturbances, or disturbances with discontinuous derivatives, to reject them completely at a plant output. That work continued under the current support, with constructive output regulators provided, controllers for hybrid output regulation of nonlinear systems, robustness principles, and the application of hybrid output regulation to the problem of spline tracking.

**Event-triggered control:** Event-triggered control is an emerging area of study, developed over the last decade, where control signal updates are made only when necessary. Assessment of the need to update a control signal can be made continuously, periodically, or intermittently. In each case, a hybrid dynamical systems model captures well the dynamical behavior of the overall system. However, the case of periodic assessment, especially for linear plans, is especially well suited for an accurate Lyapunov-function analysis employing linear matrix inequality conditions. We proposed such analysis and synthesis tools.

**Multi-agent and networked systems:** Multi-agent systems involve complicated interactions between dynamic agents, in some problems cooperating to achieve a common goal, like estimation consensus or synchronization, and other times being non-cooperative, attempting to maximize their own payoff function. Some multi-agent systems algorithms are deterministic while others employ randomness to achieve or enhance robustness. We used extremum-seeking ideas and switched system ideas for cooperative multi-agent systems, used reset concepts for robust consensus, and analyzed the role of randomness and asynchronous actions in robust Nash equilibrium seeking in non-cooperative games. We provided new analysis tools for asynchronous, nonlinear, networked control systems. We used stochastic hybrid control to achieve almost sure global synchronization on the circle under different criteria.

**Hybrid systems theory:** Hybrid dynamical systems are used to model situations where the state sometimes changes continuously and other times changes instantaneously through jumps. Such models address systems with state resets, mechanical systems with impacts, logic-based switching between controllers, networked control systems, etc. Various stability analysis tools for hybrid systems have been developed over the last twenty years, and yet there are several aspects of stability theory for hybrid systems that are not yet suitably developed. Such tools are critical for certifying effective behavior of hybrid control systems. In the current period, we provided new sufficient conditions for global asymptotic stability, a hybrid Gronwall lemma, a Lyapunov function based small-gain theorem, analysis tools for assessing finite-gain  $L_p$  stability, sufficient conditions for exponential stability, a complete characterization of input-to-state stability for hybrid systems, and new results on averaging theory for hybrid systems. Moreover, we began to develop a general framework that handles hybrid systems with delays.

**Stochastic discrete-time and hybrid systems:** Stochastic hybrid systems go beyond standard hybrid dynamical systems by permitting the evolution to be driven by random processes, providing even more modeling flexibility. Various types of stochastic hybrid systems have been studied for multiple decades. We generated a paper surveying this literature. One aspect that is particular to all of these earlier results is that uniqueness of solutions is enforced. However, there are many compelling reasons to study stochastic systems that admit multiple solutions. One motivation comes from treating open systems with disturbances as closed systems with non-unique solutions. Another motivation comes from attempting to guarantee robustness of stability for systems with non-regular data. These motivations exist even for discrete-time stochastic systems, where non-uniqueness is also not usually considered in the literature. Recognizing the importance of establishing a better understanding of stochastic discrete-time and hybrid dynamical systems, in particular for the development of effective stochastic, hybrid control algorithms, we published a wide variety of results on this topic. Our results on stochastic discrete-time (difference) inclusions include an invariance principle, converse Lyapunov theorems, equivalent characterizations of input-to-state stability, relaxed sufficient conditions for asymptotic stability and recurrence, robustness results for discontinuous stochastic discrete-time systems, and an application to robust Nash equilibrium seeking in non-cooperative games. For hybrid systems with non-stochastic flows (continuous evolution) but stochastic jumps, we provided an extensive set of sufficient conditions for asymptotic stability in probability and recurrence. We also developed a useful sequential compactness result, and an ensuing



invariance/recurrence principle. We considered hybrid systems with both stochastic flows and stochastic jumps, including Lyapunov function based sufficient conditions for asymptotic stability in probability and recurrence. Many open problems remain in completing the story on stability theory for stochastic hybrid systems with non-unique solutions. Development of these tools will enhance our ability to control complex dynamical systems using randomness and logic-based, decision-making criteria.

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### **Archival Publications (published) during reporting period:**

The research supported by this grant resulted in 24 journal papers, 25 refereed conference papers, and 2 book chapters. These publications are listed below.

#### Journal papers

J1. A.R. Teel, "A recurrence principle for stochastic difference inclusions", IEEE Transactions on Automatic Control, vol. 60, no. 2, pp. 420-435, 2015.

J2. Prieur, C.; Teel, A.R.; Zaccarian, L., "Relaxed persistent flow/jump conditions for uniform global asymptotic stability", IEEE Transactions on Automatic Control, vol. 59, no. 10, pp. 2766-2771, 2014.

J3. A.R. Teel, A. Subbaraman, and A. Sferlazza, "Stability analysis for stochastic hybrid systems: a survey", Automatica, vol. 50, no. 10, pp. 2435-2456, 2014.

J4. N. Noroozi, D. Nešić, and A.R. Teel, "Gronwall inequality for hybrid systems", Automatica, vol. 50, no. 10, pp. 2718-2722, 2014.

J5. A.R. Teel, J.P. Hespanha, and A. Subbaraman, "A converse Lyapunov theorem and robustness for asymptotic stability in probability", IEEE Transactions on Automatic Control, vol. 59, no. 9, pp. 2426-2441, 2014.

J6. Liberzon, D.; Nešić, D.; Teel, A.R., "Lyapunov-based small-gain theorems for hybrid systems", IEEE Transactions on Automatic Control, vol. 59, no. 6, pp. 1395-1210, 2014.

J7. N. Cox, L. Marconi, and A.R. Teel, "High-gain observers and linear output regulation for hybrid exosystems", International Journal of Robust and Nonlinear Control, 24 (6), 1043-1063, 2014.

J8. Teel, A.R.; Hespanha, J.P.; Subbaraman, A., "Equivalent characterizations of input-to-state stability for stochastic discrete-time systems," IEEE Transactions on Automatic Control, vol.59, no.2, pp. 516-522, 2014.

J9. Marconi, L.; Teel, A.R., "Internal model principle for linear systems with periodic state jumps", IEEE  
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Transactions on Automatic Control, vol.58, no.11, pp. 2788-2802, 2013.

J10. Mayhew, C.G.; Teel, A.R., "Synergistic hybrid feedback for global rigid-body attitude tracking on  $SO(3)$ ", IEEE Transactions on Automatic Control, vol.58, no.11, pp.2730-2742, 2013.

J11. A. Subbaraman, A.R. Teel, "A Matrosov theorem for strong global recurrence", Automatica, Volume 49, Issue 11, November 2013, Pages 3390-3395.

J12. A. Subbaraman, A.R. Teel, "A converse Lyapunov theorem for strong global recurrence", Automatica, Volume 49, Issue 10, October 2013, Pages 2963-2974.

J13. S. Grammatico, A. Subbaraman, A.R. Teel, "Discrete-time stochastic control systems: a continuous Lyapunov function implies robustness to strictly causal perturbations", Automatica, Volume 49, Issue 10, October 2013, Pages 2939-2952.

J14. L. Fagiano, A.R. Teel, "Generalized terminal state constraint for model predictive control", Automatica, Volume 49, Issue 9, September 2013, Pages 2622-2631.

J15. D. Nešić, A.R. Teel, G. Valmorbida, L. Zaccarian, "Finite-gain  $L_p$  stability for hybrid dynamical systems", Automatica, Volume 49, Issue 8, August 2013, Pages 2384-2396.

J16. Teel, A.R., "A Matrosov theorem for adversarial Markov decision processes", IEEE Transactions on Automatic Control, vol.58, no.8, pp.2142-2148, 2013.

J17. C. G. Mayhew, A. R. Teel, "Global stabilization of spherical orientation by synergistic hybrid feedback with application to reduced-attitude tracking for rigid bodies", Automatica, Volume 49, Issue 7, July 2013, Pages 1945-1957.

J18. Forni, F.; Teel, A.R.; Zaccarian, L., "Follow the bouncing ball: global results on tracking and state estimation with impacts", IEEE Transactions on Automatic Control, vol.58, no.6, pp.1470-1485, 2013.

J19. Teel, A.; Forni, F.; Zaccarian, L., "Lyapunov-based sufficient conditions for exponential stability in hybrid systems", IEEE Transactions on Automatic Control, vol.58, no.6, pp.1591-1596, 2013.

J20. Mayhew, C.G.; Sanfelice, R.G.; Teel, A.R., "On path-lifting mechanisms and unwinding in quaternion-based attitude control", IEEE Transactions on Automatic Control, vol.58, no.5, pp.1179-1191, 2013.

J21. Heemels, W.P.M.H.; Donkers, M.C.F.; Teel, A.R., "Periodic event-triggered control for linear systems", IEEE Trans. on Automatic Control, vol.58, no.4, pp. 847-861, 2013.

J22. A. R. Teel, "Lyapunov conditions certifying stability and recurrence for a class of stochastic hybrid systems", Annual Reviews in Control, Volume 37, Issue 1, April 2013, Pages 1-24.

J23. C. Cai and A.R. Teel, "Robust input-to-state stability for hybrid systems", SIAM Journal on control and optimization (2013) volume: 51 issue: 2 page: 1651-1678.

J24. R.H. Gielen, A.R. Teel, M. Lazar, "Tractable Razumikhin-type conditions for input-to-state stability analysis of delay difference inclusions", Automatica, Volume 49, Issue 2, February 2013, Pages 619-625.

#### Conference papers

C1. J.I. Poveda and A.R. Teel, "Flexible Nash seeking using stochastic difference inclusions", Proceedings of the American Control Conference, 2015, to appear.

- C2. F. Forte, L. Marconi, A.R. Teel, "Robust nonlinear regulation: continuous-time internal models and hybrid identifiers", Proceedings of the 53rd IEEE Conference on Decision and Control, 4703-4708, 2014.
- C3. C.O. Saglam, A.R. Teel, and K. Byl, "Lyapunov-based versus Poincare map analysis of the rimless wheel", Proceedings of the 53rd IEEE Conference on Decision and Control, 1514-1520, 2014.
- C4. A.R. Teel, "Stochastic hybrid inclusions with diffusive flows", Proceedings of the 53rd IEEE Conference on Decision and Control, 3071-3076, 2014.
- C5. J.I. Poveda and A.R. Teel, "A hybrid seeking approach for robust learning in multi-agent systems", Proceedings of the 53rd IEEE Conference on Decision and Control, 3463-3468, 2014.
- C6. M. Hartman and A.R. Teel, "Adding resets to an average consensus algorithm to achieve practical input-to-output stability", Proceedings of the 53rd IEEE Conference on Decision and Control, 704-708, 2014.
- C7. S.J.L.M. van Loon, W.P.M.H. Heemels, and A.R. Teel, "Improved L2-gain analysis for a class of hybrid systems with applications to reset and event-triggered control", Proceedings of the 53rd IEEE Conference on Decision and Control, pp. 1221-1226, 2014.
- C8. A. Subbaraman and A.R. Teel, "A Krasovskii-LaSalle function based recurrence principle for a class of stochastic hybrid systems", Proceedings of the 53rd IEEE Conference on Decision and Control, pp. 2310-2315, 2014.
- C9. A.R. Teel, "On sequential compactness of solutions for a class of stochastic hybrid systems", Proceedings of the American Control Conference, pp. 4512-4517, 2014.
- C10. A.R. Teel, "On a recurrence principle for a class of stochastic hybrid systems", Proceedings of the American Control Conference, pp. 4518-4523, 2014.
- C11. J.T. Isaacs, C. Magee, A. Subbaraman, F. Quitin, K. C. Fregene, A. R. Teel, U. Madhow, J.P. Hespanha, "GPS-optimal micro air vehicle navigation in degraded environments", Proceedings of the American Control Conference, 1864-1871, 2014.
- C12. N. Cox, L. Marconi, and A.R. Teel, "Design of robust internal models for a class of linear hybrid systems", IFAC World Congress, 19 (1), 1525-1530, 2014.
- C13. J. Liu and A.R. Teel, "Hybrid systems with memory: modeling and stability analysis via generalized solutions", IFAC World Congress, 19 (1), 6019-6024, 2014.
- C14. A.H. Brodtkorb, A.J. Sorensen, and A.R. Teel, "Increasing the operation window of dynamic positioned vessels using the concept of hybrid control", ASME 33rd International Conference on Ocean, Offshore and Arctic Engineering, 2014.
- C15. Heemels, W.P.M.H.; Borgers, D.P.; van de Wouw, N.; Nešić, D.; Teel, A.R., "Stability analysis of nonlinear networked control systems with asynchronous communication: A small-gain approach," 52nd IEEE Conference on Decision and Control, pp.4631-4637, 2013.
- C16. Subbaraman, A.; Hartman, M.; Teel, A.R., "A stochastic hybrid algorithm for robust global almost sure synchronization on the circle: all-to-all communication," 52nd IEEE Conference on Decision and Control, pp.600-605, 2013.

C17. Grammatico, S.; Subbaraman, A.; Teel, A.R., "Discrete-time stochastic control systems: examples of robustness to strictly causal perturbations," 52nd IEEE Conference on Decision and Control, pp.6403-6408, 2013.

C18. Teel, A.R.; Goebel, R.; Morris, B.; Ames, A.D.; Grizzle, J.W., "A stabilization result with application to bipedal locomotion," 52nd IEEE Conference on Decision and Control, pp.2030-2035, 2013.

C19. Cox, N.; Marconi, L.; Teel, A.R., "Results on non-linear hybrid output regulation," 52nd IEEE Conference on Decision and Control, pp. 2036-2041, 2013.

C20. A.R. Teel. "Sure almost global vs. global almost sure synchronization on the circle: the virtues of stochastic hybrid feedback", Proceedings of the 9th IFAC Symposium on Nonlinear Control Systems (NOLCOS 2013), pp. 217-222, Sept. 2013.

C21. Liberzon, D.; Nešić, D.; Teel, A.R., "Small-gain theorems of LaSalle type for hybrid systems," Decision and Control (CDC), IEEE 51st Annual Conference on, pp.6825-6830, 2012.

C22. W. Wang; Teel, A.R.; Nešić, D., "Averaging in singularly perturbed hybrid systems with hybrid boundary layer systems," Decision and Control (CDC), IEEE 51st Annual Conference on, pp.6855-6860, 2012.

C23. Teel, A.R.; Hespanha, J.; Subbaraman, A., "Stochastic difference inclusions: results on recurrence and asymptotic stability in probability," Decision and Control (CDC), IEEE 51st Annual Conference on, pp.4051-4056, 2012.

C24. Cox, N.; Marconi, L.; Teel, A.R., "Hybrid internal models for robust spline tracking," Decision and Control (CDC), IEEE 51st Annual Conference on, pp. 4877-4882, 2012.

C25. Jun Liu; Teel, A.R., "Generalized solutions to hybrid systems with delays," Decision and Control (CDC), IEEE 51st Annual Conference on, pp. 6169-6174, 2012.

#### Book Chapters

B1. M. Hartman, A. Subbaraman, A.R. Teel. Robust global almost sure synchronization on a circle via stochastic hybrid control, In Control of cyber-physical systems: Workshop Held at Johns Hopkins University, April 2013, Springer, pp. 3-21.

B2. F. Forni, A.R. Teel, and L. Zaccarian, "Reference mirroring for control with impacts", In Hybrid Systems with Constraints, pp. 213-255, Wiley, 2013.

#### Changes in research objectives (if any):

None

#### Change in AFOSR Program Manager, if any:

None

#### Extensions granted or milestones slipped, if any:

None

#### AFOSR LRIR Number

#### LRIR Title

#### Reporting Period

#### Laboratory Task Manager

#### Program Officer

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**Research Objectives**

**Technical Summary**

**Funding Summary by Cost Category (by FY, \$K)**

|                      | Starting FY | FY+1 | FY+2 |
|----------------------|-------------|------|------|
| Salary               |             |      |      |
| Equipment/Facilities |             |      |      |
| Supplies             |             |      |      |
| Total                |             |      |      |

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